62. **Answer (D):** For any x value $|-2x| = 2|x|$, $-|3x| = -3|x|$, so the first term is $2|x| - 3|x| = -|x|$. Similarly $|3x| = 3|x|$ and $|-5x| = 5|x|$, so the second term is simply $-|3|x| - 5|x|| = -|-2|x|| = -2|x|$. The sum is then $-|x| - 2|x| = -3|x|$.

63. **Answer (A):** Only (A) is correct. By definition of M, (A) is equivalent to:

$$(x1 + x2 + x3 + x4)/4 = ((x1 + x2)/2 + (x3 + x4)/2)/2,$$

establishing the identity. Using $x_1 = 1$, $x_2 = 2$, $x_3 = 3$ and $x_4 = 4$ suffices to show the other four statements are not true.

64. **Answer (C):** Since the minor axis of the ellipsoid is 2, the sphere's diameter is also 2. If the edge of the cube is a, then the largest diagonal is $\sqrt{3}a$ and that is equal to the sphere's diameter. So, $a = 2\sqrt{3}3$ and the volume of the cube is $\dfrac{8\sqrt{3}}{9}$.

65. *Answer: 1216* Since the perimeters of the quadrilaterals are equal and each is equilateral, the sides are equal. Let x be the length of a side. The areas of the two quadrilaterals are: $x^2 \sin\angle ABC$ and $x^2 \sin\angle EFG$ respectively.

$$\begin{aligned} x^2 \sin\angle EFG - x^2 \sin\angle ABC &= 64 \\ x^2(\sin\angle EFG - \sin\angle ABC) &= 64 \\ x(0.05) &= 64 \\ x^2 &= 1280 \end{aligned}$$

Therefore the area of the square is 1280 and the area of $ABCD$ is $1280 - 64 = 1216$.

66. **Answer (C):** The portion of the each cube removed is a pyramid with an isosceles right triangular base of area 1/2 and height 1, so its volume is 1/6. Therefore the remaining portion of each cube has volume 5/6, so $v = 10/6$. From the original surface area of 6 for each cube, three triangles of area 1/2 each have been removed, so $a = 9$. Therefore $a/v = 27/5$.

67. *Answer: 19* Straightforward arithmetic reveals that $(f_1(9), f_2(9), \ldots, f_{18}(9), f_{19}(9)) =$

$$(28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1).$$

Therefore 1 first appears as $f_{19}(9)$.

Comment: It is an unsolved problem in mathematics whether for every positive integer n there exists a k such that $f_k(n) = 1$.

68. :Answer: 39213 The area of the triangle is $A = 2012 \cdot 40/2 = 40240$. There are $2012 + 40 + 1 = 2053$ lattice points on the horizontal and vertical sides of the

triangle, and because $\gcd(2012, 40) = 4$, the only lattice points in the interior of the third side are $(503, 30)$, $(1006, 20)$, and $(1509, 10)$, for a total of $B = 2056$ lattice points on the boundary of the triangular region. By Pick's formula,

$$A = I + \frac{B}{2} - 1,$$

where I is the number of lattice points in the interior. Solving for I gives 39213.

69. **Answer (C):** Note that $1 + 2\cos\beta + \cos^2\beta - 2\cos\beta + \sin^2\beta = 1 + \cos^2\beta + \sin^2\beta = 1 + 1 = 2$. Thus the answer is (C).

70. **Answer (E):** One of the diagonals of the rhombus has slope $\dfrac{b-f}{a-e}$, and this slope is 10 by the given logarithm condition. The perpendicular diagonal, which has slope $\dfrac{d-h}{c-g}$, must therefore have slope $\dfrac{-1}{10}$. Thus $\dfrac{d-h}{c-g} = \dfrac{2}{c-g} = \dfrac{-1}{10}$, from which we get $g - c = 20$.

71. *Answer: 18* Let the trapezoid be $ABCD$, where $AB = \log 3$ and $CD = \log 192$. Let E and F be the respective feet of the perpendiculars from A and B to $\overline{CD}$. Because $ABCD$ is isosceles, $DE = CF = \frac{CD-EF}{2} = \frac{1}{2}(\log 192 - \log 3) = \frac{1}{2}\log 64 = \log 8 = 3\log 2$. Note that ADE is a right triangle with legs $3\log 2$ and $4\log 2$, so by the Pythagorean Theorem, $AD = BC = 5\log 2$. Thus the perimeter of $ABCD$ is $\log 3 + \log 192 + 2(5\log 2) = 2\log 3 + 16\log 2 = \log 2^{16}3^2$. Thus $p + q = 16 + 2 = 18$.